Generalized Parton Distributions and the Dependence of Parton Distributions on the Impact Parameter

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Abstract. Generalized parton distributions (GPDs) provide a link between form factors, parton distributions and other observables. I discuss the connection between GPDs and parton distributions as a function of the impact parameter. Since this connection involves GPDs in the limit of vanishing skewedness parameter ξ, i.e. when the off-forwardness is purely transverse, I also illustrate how to relate $\xi \neq 0$ data to $\xi = 0$ data, which is important for experimental measurements of these observables.

INTRODUCTION

Deeply virtual Compton scattering experiments provide a useful tool for probing off-forward or generalized parton distributions (GPDs) [1,2]

$$\bar{p}^+ \int \frac{dx^-}{2\pi} \langle p' | \bar{q}(\frac{-x^-}{2})\gamma^+ q(\frac{x^-}{2}) | p \rangle e^{ix\bar{p}^+ x^-} = H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) \qquad (1)$$

$$+ E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p),$$

$$\bar{p}^+ \int \frac{dx^-}{2\pi} \langle p' | \bar{q}(\frac{-x^-}{2})\gamma^+\gamma_5 q(\frac{x^-}{2}) | p \rangle e^{ix\bar{p}^+ x^-} = \tilde{H}^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) \qquad (2)$$

$$+ \tilde{E}^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p),$$

where $x^\pm = x^0 \pm x^3$ and $p^+ = p^0 + p^3$ refer to the usual light-cone components, $\bar{p} = \frac{1}{2}(p+p')$, $\Delta = p - p'$, and $t \equiv \Delta^2$. The skewedness in Eqs. (1,2) is defined as $\xi \equiv \frac{\Delta^+}{p^+}$. From the point of view of parton physics in the infinite momentum frame (IMF), GPDs have the physical meaning of the amplitude for the process that a quark is taken out of the nucleon with longitudinal momentum fraction x and then inserted back into the nucleon with a four momentum transfer Δ^μ [3]. GPDs play multiple roles and in a certain sense they interpolate between form factors

and conventional parton distributions (PDs) [1,2]: for $\xi = t = 0$ one recovers conventional PDs, i.e. longitudinal momentum distributions in the IMF, while when one integrates $H^q(x, \xi, t)$ over x, one obtains a form factor, i.e. the Fourier transform of a position space density (in the Breit frame!). One of the new physics insights that one can learn from these GPDs is the angular momentum distribution [4]. Others include meson distribution amplitudes. [1]

In this note, we will discuss the limit $\xi \to 0$, but $t \neq 0$, i.e. when the momentum transfer is purely transverse. In this limit, the "E-terms" in Eqs. (1) and (2) drop out and one finds

$$\int \frac{dx^-}{4\pi} \langle p'|\bar{q}(\frac{-x^-}{2})\gamma^+ q(\frac{x^-}{2})|p\rangle e^{ixp^+x^-} = H^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \tag{3}$$

$$\int \frac{dx^-}{4\pi} \langle p'|\bar{q}(\frac{-x^-}{2})\gamma^+ \gamma_5 q(\frac{x^-}{2})|p\rangle e^{ix\bar{p}^+x^-} = \tilde{H}^q(x, 0, -\boldsymbol{\Delta}_\perp^2), \tag{4}$$

with $p^{+\prime} = p^+$. Eqs. (3) and (4) very much resemble the definitions for ordinary twist-2 PDs, with the only difference being the fact that the $\perp$ momenta of the initial and final state are not the same. The situation here is very analogous to the relation between the forward and off-forward matrix elements of a current, i.e. between a charge and charge form factor. The main difference is of course that the operator entering the 'form factor' in Eq. (4) is not the current operator, but the operator that measures longitudinal momentum distributions. From this analogy, and since charge form factors have the physical interpretation of the Fourier transform of the position space charge distribution, it is natural to expect a similar interpretation also for GPDs.

GPDS FOR $\xi = 0$

In the following we will use a light-front (LF) Fock expansion to represent GPDs for $\xi = 0$ as overlap integrals between LF wave functions $\Psi_N(x, \mathbf{k}_\perp)$ summed ($\sum_N$) over Fock components [6] [2]

$$H^q(x, 0, -\boldsymbol{\Delta}_\perp^2) = \sum_N \sum_j \int [dx]_N \int \left[d^2\mathbf{k}_\perp\right]_N \delta(x - x_j)\Psi_N^*(x, \mathbf{k}'_\perp)\Psi_N(x, \mathbf{k}_\perp), \tag{5}$$

where $\sum_j$ denotes the sum over all quarks with flavor q in that Fock component and $\mathbf{k}'_{\perp,i} = \mathbf{k}_{\perp,i} - x_i\boldsymbol{\Delta}_\perp$ for $i \neq j$, while $\mathbf{k}'_{\perp,j} = \mathbf{k}_{\perp,j} + (1 - x_j)\boldsymbol{\Delta}_\perp$. Upon switching to the coordinate representation in the $\perp$ direction

$$\Psi_N(x, \mathbf{k}_\perp) = \int \frac{[d\mathbf{b}_\perp]}{(2\pi)^N} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}_\perp} \tilde{\Psi}_N(x, \mathbf{b}_\perp) \tag{6}$$

it is straightforward to see that

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[1] For a discussion of this connection in the context of QCD_{1+1} see Ref. [5].
[2] Note that this is an exact expression provided one knows the Ψ_N for *all* Fock components

$$H^q(x, 0, -\mathbf{\Delta}_\perp^2) = \int [dx] \int [d\mathbf{b}_\perp] \sum_N \sum_j \delta(x - x_j) e^{i\mathbf{\Delta}_\perp \cdot (\mathbf{b}_{\perp,j} - \mathbf{R}_\perp)} \left| \tilde{\Psi}_N(x, \mathbf{b}_\perp) \right|^2. \quad (7)$$

where we have introduced the $\perp$ *center of momentum*

$$\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{b}_{\perp,i}.. \quad (8)$$

Eq. (7) illustrates that GPDs for $\xi = 0$ can be interpreted as Fourier transforms of *impact parameter dependent PDs*

$$H(x, 0, -\mathbf{\Delta}_\perp^2) = \int d^2\mathbf{r}_\perp q(x, \mathbf{r}_\perp) e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{r}_\perp}$$

$$\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) = \int d^2\mathbf{r}_\perp \Delta q(x, \mathbf{b}_\perp) e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{r}_\perp} \quad (9)$$

where for example (again we suppress spin indices for simplicity)

$$q(x, \mathbf{r}_\perp) = \sum_N \sum_j \int [dx] \int [d\mathbf{b}_\perp] \delta(x - x_j) \delta(\mathbf{r}_\perp - (\mathbf{b}_{\perp,j} - \mathbf{R}_\perp)) \left| \tilde{\Psi}_N(x, \mathbf{b}_\perp) \right|^2. \quad (10)$$

Thus, GPDs, in the limit of $\xi \to 0$, allow a simultaneous determination of the longitudinal momentum fraction and transverse impact parameter of partons in the target hadron in the IMF.

Eq. (9) is not only a re-derivation of the main result from Ref. [7] using LF Fock wave functions, but it also clearly illustrates that the impact parameter in the impact parameter dependent PDs entering Eq. (9) is measured w.r.t. $\mathbf{R}_\perp$. There is a striking similarity between this observation and the fact that the Fourier transform of form factors in nonrelativistic (NR) systems yields charge distributions measured w.r.t. $\vec{R}_{CM} = \sum_i m_i \vec{r}_i / M$. This should not come as a surprise, since there is a residual Galilei invariance under the purely kinematic $\perp$ boosts in the LF framework

$$x_i \longrightarrow x_i' = x_i \quad , \qquad \mathbf{k}_{i\perp} \longrightarrow \mathbf{k}_{i\perp}' \equiv \mathbf{k}_{i\perp} + x_i \mathbf{\Delta P}_\perp, \quad (11)$$

which resembles very much NR boosts

$$\vec{k}_i \longrightarrow \vec{k}_i' = \vec{k}_i + m_i \Delta \vec{v} = \vec{k}_i + \frac{m_i}{M} \Delta \vec{P}. \quad (12)$$

The above observation about the $\perp$ center of momentum has one immediate consequence for the $x \to 1$ behavior of $q(x, \mathbf{b}_\perp)$. Since the weight factors in the definition of $\vec{R}_\perp$ are the momentum fractions, any parton i that carries a large fraction x_i of the target's momentum will necessarily have a $\perp$ position $\vec{r}_{i\perp}$ that is close to $\vec{R}_\perp$. Therefore the transverse profile (i.e. the dependence on $\mathbf{b}_\perp$) of $q(x, \mathbf{b}_\perp)$ will necessarily become more narrow as $x \to 1$, i.e. we expect that partons become very localized in $\perp$ position as $x \to 1$. By Fourier transform, this also implies that the slope of $H(x, 0, t)$ w.r.t. t at $t = 0$, i.e.

$$\langle \vec{b}_\perp^2 \rangle \equiv 4 \frac{\frac{d}{dt} H(x, 0, t)|_{t=0}}{H(x, 0, 0)} \quad (13)$$

should in fact vanish for $x \to 1$!

EXTRAPOLATING TO $\xi \to 0$

From the experimental point of view, $\xi = 0$ is not directly accessible in DVCS since one needs some longitudinal momentum transfer in order to convert a virtual photon into a real photon. There are several ways around this difficulty. First of all, one can access $\xi = 0$ in real wide angle Compton scattering [8]. However, it should also be possible to perform DVCS experiments at finite ξ and to extrapolate to $\xi = 0$. This extrapolation is greatly facilitated by working with moments since the ξ dependence of the moments of GPDs is in the form of polynomials [3]. For example, for the even moments $H_n(\xi, t) \equiv \int_{-1}^{1} dx x^{n-1} H(x, \xi, t)$ one finds [4]

$$H_n(\xi, t) = A_{n,0}(t) + A_{n,2}(t)\xi^2 + ... + A_{n,n-2}(t)\xi^{n-2} + C_n(t)\xi^n, \tag{14}$$

i.e. for example

$$\int_{-1}^{1} dx x H(x, \xi, t) = A_{2,0}(t) + C_2(t)\xi^2. \tag{15}$$

Since the H_n have a known functional dependence on ξ, one can use measurements of the moments of GPDs at nonzero values of ξ to determine (fit) the "form factors" $A_{n,2i}(t)$ and $C(t)$. After determining these invariant form factors, one can evaluate Eq. (14) for $\xi = 0$, yielding $H_n(0, t) = A_{n,0}(t)$, and the impact parameter dependence of the $n - th$ moment of the parton distribution in the target reads

$$q_n(\mathbf{b}_\perp) \equiv \int_{-1}^{1} dx x^{n-1} q(x, \mathbf{b}_\perp) = \int d^2 q_\perp A_{n,0}(-\boldsymbol{\Delta}_\perp^2) e^{i\boldsymbol{\Delta}_\perp \mathbf{b}_\perp}. \tag{16}$$

A very similar procedure can be applied to the moments of spin dependent GPDs

$$\tilde{H}_n(\xi, t) \equiv \int_{-1}^{1} dx x^{n-1} \tilde{H}(x, \xi, t) = \tilde{A}_{n,0}(t) + \tilde{A}_{n,2}(t)\xi^2 + ... + \tilde{A}_{n,n-1}(t)\xi^{n-1}. \tag{17}$$

Similarly in the unpolarized case, one can extract the $\frac{n+1}{2}$ form factors of the nth moment from measurements of $\tilde{H}$ for $\frac{n+1}{2}$ different values of ξ (and the same values of t), yielding for the impact parameter dependence of the nth moment of the polarized PD

$$\Delta q_n(\mathbf{b}_\perp) \equiv \int_{-1}^{1} dx x^{n-1} \Delta q(x, \mathbf{b}_\perp) = \int d^2 \boldsymbol{\Delta}_\perp \tilde{A}_{n,0}(-\boldsymbol{\Delta}_\perp^2) e^{i\boldsymbol{\Delta}_\perp \mathbf{b}_\perp}. \tag{18}$$

Of course, this procedure becomes rather involved for high moments, but the steps outlined above seem to be a viable way of determining the impact parameter dependence of low moments of parton distributions from DVCS data.

SUMMARY AND OUTLOOK

GPDs for $\xi \to 0$, i.e. where the off-forwardness is only in the $\perp$ direction, can be identified with the Fourier transform of impact parameter dependent PDs. Here the impact parameter $\mathbf{b}_\perp$ is defined as the $\perp$ distance from the center of (longitudinal) momentum in the IMF. This identification of GPDs with Fourier transforms of impact parameter dependent PDs is very much analogous to the identification of the charge form factor with the Fourier transform of a charge distribution in position space.

Although the $\xi \to 0$ limit of GPDs cannot be probed directly in DVCS, one can use the know polynomial ξ-dependence of the x-moments to extrapolate to $\xi = 0$.

Knowing the impact parameter dependence allows one to gain information on the spatial distribution of partons inside hadrons and to obtain new insights about the nonperturbative intrinsic structure of hadrons. For example, the pion cloud of the nucleon is expected to contribute more for large values of $\mathbf{b}_\perp$. Shadowing of small x parton distributions, is probably stronger at small values of $\mathbf{b}_\perp$ since partons in the geometric center of the nucleon are more effectively shielded by the surrounding partons than partons far away from the center. Geometric models for the small x behavior of the PDs in the nucleon suggest that polarized PDs may be more spread out in $\mathbf{b}_\perp$ than unpolarized ones [9]. These and many other models and intuitive pictures for the parton structure of hadrons give rise to predictions for the impact parameter dependence of PDs that reflect the underlying microscopic dynamics of these models. Our results may also play an important role in the modeling of the t dependence in GPDs, which may in turn be relevant for fitting GPDs to experimental data for DVCS amplitudes. Finally, combining information about the impact parameter dependence with information about longitudinal correlations in position space [10] may lead to further insights into non-perturbative hadron structure.

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